Nonstationary Seismic Response Analysis of Long-span Structures by Frequency Domain Method Considering Wave Passage Effect

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Abstract

In this paper, a frequency domain method is proposed for the nonstationary seismic analysis of long-span structures subjected to random ground motions considering the wave passage effect. Based on the correlation analysis theory and fast Fourier transform (FFT), a semi-analytical solution is derived for the evolutionary power spectral density of the random response of long-span structures in the frequency domain. The expression of this solution indicates that the evolutionary property of nonstationary random responses can be determined completely by the modulation function of random ground motions, and hence the solution has clear physical interpretations. For slowly varying modulation functions, the FFT can be implemented with a small sampling frequency, so the present method is very efficient within a given accuracy. In numerical examples, nonstationary random responses of a long-span cable stayed bridge to random ground motions with the wave passage effect are studied by the present method, and comparisons are made with those of the pseudo excitation method (PEM) to verify the present method. Then the accuracy and efficiency of the present method with different sampling frequencies are compared and discussed. Finally, the influences of the apparent velocity of the seismic waves on nonstationary random responses are investigated.

Key words: seismic analysis; wave passage effect; nonstationary; evolutionary power spectral density; frequency domain method

1 Introduction

During an earthquake, the energy released at the epicenter transfers to the ground surface in the form of seismic waves. Since the waves travel along different paths and through a complex medium, ground motions caused by the earthquake at different locations will have significant differences. Even if the propagation medium is exactly uniform, there is still a difference in the arrival times of seismic waves at different locations due to their different distances to the epicenter. This phenomenon is known as the “wave passage effect”. Long-span structures are generally important facilities, e.g. long-span bridges, dams, or nuclear power plants. Therefore, their aseismatic capabilities are highly relevant to public safety. In seismic analysis, long-span structures have their own special features compared to general building structures. A major feature is that these structures extend over long distances parallel to the ground, so their supports undergo different motions during an earthquake. Hence, the dynamic behaviors of long-span structures with and without consideration of the wave passage effect have significant differences [1, 2].

The time-history method is widely applied for the random analysis of long-span structures subjected to an earthquake with spatial variation [3]. This method is based on stochastic simulation, and response parameters (mainly mean values and variances) are obtained through statistical analysis of samples of the random responses. Its main drawback, however, is that it has a huge computational cost. Over three decades, some more efficient methods have been developed. One of them is an extension of the conventional response spectrum method, which was initially only feasible for uniform seismic excitation. Der Kiureghian and Neuenhofer [4] developed a special response spectrum method for the response of structures to a random earthquake considering the wave passage effect, incoherence effect and site-response effect. Yamamura and Tanaka [5] presented an analysis of a suspension bridge to multi-support seismic excitations. In their work, ground motions within a group of adjacent supports on continuous soil or rock were assumed to be uniform and synchronized, while those of different groups were treated as non-uniform and uncorrelated. Berrah and Kausel [6] proposed a modified response spectrum method to address the problem of long-span structures subjected to imperfectly correlated seismic excitations. However, they did not consider the influence of quasi-static displacement. Due to the naturally random properties of the earthquake, it is more rational to study the seismic response of long-span structures using random vibration theory. Heredia-Zavoni and Vanmarcke [7] developed a random vibration method for the seismic analysis of linear multi-support systems. This method reduced the response evaluation to that of a series of linear one degree systems in a way that fully accounts for the space-time correlation structure of the ground motion. Lee and Penzien [8] studied random responses of piping systems under multi-support excitations, obtaining mean and extreme values of the systems in either the time or the frequency domain. Lin et al. [9] simplified a surface-mounted pipeline as an infinitely long

Bernoulli-Euler beam attached to evenly spaced ground supports, and solved its random seismic responses. Zanardo et al. [10] carried out a parametric study of the pounding phenomenon associated with the seismic response of multi-span simply supported bridges with base isolation devices. Tubino et al. [11] investigated the influence of the partial correlation of the seismic ground motion on long-span structures by introducing suitable equivalent spectra. Lupoi et al. [12] studied the effects of the spatial variation of ground motion on the response of bridge structures. The results showed that the spatial variation affects the random response considerably. Lin et al. [13-14] proposed a random vibration method known as the pseudo-excitation method (PEM). In the framework of the PEM, the random vibration analysis was reduced to relatively simple harmonic or transient analysis, and hence its computation was of high efficiency. The PEM was also used for seismic responses of long-span structures to ground motion with spatial variations.

In the research mentioned above, ground motions were always assumed to be stationary random processes. However, some practical observation results showed that the intensity of the ground motion had three obvious stages, i.e. increasing, steady and decreasing, during the duration of the earthquake. Hence it is more rational to assume the ground motion as a nonstationary random process. Spectral methods, such as Wigner-Ville spectrum [15], physical spectrum [16], evolutionary spectrum [17,18] etc., can provide a general description of the energy-frequency properties of nonstationary processes, and thus have been a focal point of study. The evolutionary power spectral density (PSD) was widely used in the earthquake engineering for its clear physical interpretation and relatively simple mathematical derivation [19,20]. An evolutionary PSD is always defined as the product of a deterministic uniform or nonuniform modulation function and a stationary PSD. Based on a spectral representation based simulation algorithm, Deodatis [21] introduced an iterative scheme to generate seismic ground motion samples at several locations on the ground surface that were compatible with prescribed response spectra, correlated according to a given coherence function, include the wave passage effect. Alderucci and Muscolino [22] presented a random vibration analysis of linear classically damped structural systems subjected to fully nonstationary multicorrelated excitations and gave a closed-form solution of the evolutionary PSD of the response. Combining the experimental data of a multi-support seismic shaking table test and structural health monitoring findings, Ozer et al. [23] developed a framework to evaluate random seismic response and estimate reliability of bridges under multi-support excitations. In the authors' previous works [13,24], the PEM and a highly accurate step-by-step integration method named the Precise Integration Method (PIM) were combined to solve nonstationary random responses of long-span structures under the earthquake with consideration of the wave passage effect. Generally, a time-frequency domain analysis is required to obtain the solution of the evolutionary PSD when structures are excited by a nonstationary random excitation. During the time-frequency domain analysis, the time domain integration is performed at each frequency point. To achieve accurate results, small time steps are required in the time domain integration, especially for a wide band random excitation with high frequency components. Hence, there will inevitably be a huge computational cost.

Combining the evolutionary PSD and correlation analysis theory, this paper develops a frequency domain method for the random vibration analysis of long-span structures subjected to ground motions with the wave passage effect. This method can be used to obtain the semi-analytical solution of the evolutionary PSD of random responses and its computation is very efficient. This paper is structured as follows. In section 2, governing equations of long-span structures subjected to nonuniform earthquake excitation are given. Section 3 presents the evolutionary PSD model with consideration of the wave passage effect. By separating the deterministic modulation function from the evolutionary PSD, section 4 establishes a frequency domain method to obtain the semi-analytical solution of random responses. In section 5, a long-span cable-stayed bridge is adopted as an example structure. The present method is applied to random vibration analysis of the bridge and the results are compared to those of the PEM to verify the present method. The influences of the wave velocity on random responses are compared and discussed. Section 6 gives some conclusions.

2 Governing equations of structures under nonuniform seismic excitation

The governing equations of a long-span structure with N supports and n degrees of freedom (DOF) subjected to nonuniform seismic excitation can be written as [25]

$$\begin{bmatrix} \mathbf{M}_{aa} & \mathbf{M}_{ab} \\ \mathbf{M}_{ab}^{\mathrm{T}} & \mathbf{M}_{bb} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{y}}_a(t) \\ \ddot{\mathbf{y}}_b(t) \end{Bmatrix} + \begin{bmatrix} \mathbf{C}_{aa} & \mathbf{C}_{ab} \\ \mathbf{C}_{ab}^{\mathrm{T}} & \mathbf{C}_{bb} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{y}}_a(t) \\ \dot{\mathbf{y}}_b(t) \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_{aa} & \mathbf{K}_{ab} \\ \mathbf{K}_{ab}^{\mathrm{T}} & \mathbf{K}_{bb} \end{bmatrix} \begin{Bmatrix} \mathbf{y}_a(t) \\ \mathbf{y}_b(t) \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{p}_b(t) \end{Bmatrix} \tag{1}$$

where the subscripts “a” and “b” indicate the non-support and support DOF, respectively; $\mathbf{y}_a(t)$ is an n-dimensional vector containing all non-support displacements; m-dimensional vectors $\mathbf{y}_b(t)$ and $\mathbf{p}_b(t)$ represent the enforced support displacements and forces at all supports, respectively; the $n \times n$ matrices $\mathbf{M}_{aa}$, $\mathbf{C}_{aa}$ and $\mathbf{K}_{aa}$ [$\mathbf{M}_{bb}$, $\mathbf{C}_{bb}$ and $\mathbf{K}_{bb}$] are the mass, damping and stiffness matrices associated with $\mathbf{y}_a(t)$ [$\mathbf{y}_b(t)$]; the superscript “T” denotes transposition. Note that when the lumped mass matrix approximation is adopted, $\mathbf{M}_{ab}$ is null.

In order to solve Eq. (1), the absolute displacement $\mathbf{y}_a(t)$ can be decomposed into the following two parts [25]:

$$\begin{Bmatrix} \mathbf{y}_a(t) \\ \mathbf{y}_b(t) \end{Bmatrix} = \begin{Bmatrix} \mathbf{y}_s(t) \\ \mathbf{y}_b(t) \end{Bmatrix} + \begin{Bmatrix} \mathbf{y}_d(t) \\ \mathbf{0} \end{Bmatrix} \tag{2}$$

in which $\mathbf{y}_s(t)$ and $\mathbf{y}_d(t)$ are the quasi-static and dynamic displacement vectors, respectively, which satisfy the following equations:

$$\begin{bmatrix} \mathbf{K}_{aa} & \mathbf{K}_{ab} \\ \mathbf{K}_{ab}^{\mathrm{T}} & \mathbf{K}_{bb} \end{bmatrix} \begin{Bmatrix} \mathbf{y}_s(t) \\ \mathbf{y}_b(t) \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{p}_b(t) \end{Bmatrix} \tag{3}$$

Expanding the first row of Eq. (3) gives

$$\mathbf{y}_s(t) = -\mathbf{K}_{aa}^{-1}\mathbf{K}_{ab}\mathbf{y}_b(t) \tag{4}$$

Assuming that the damping force is proportional to the dynamic relative velocity $\dot{\mathbf{y}}_d(t)$ instead of $\dot{\mathbf{y}}_a(t)$, the first row of Eq. (1) can be rewritten as

$$\mathbf{M}_{aa}\ddot{\mathbf{y}}_d(t) + \mathbf{C}_{aa}\dot{\mathbf{y}}_d(t) + \mathbf{K}_{aa}\mathbf{y}_d(t) = \mathbf{M}_{aa}\mathbf{K}_{aa}^{-1}\mathbf{K}_{ab}\ddot{\mathbf{y}}_b(t) \tag{5}$$

In the random vibration analysis of long-span structures under nonuniform seismic excitation, seismic waves are always assumed to travel along a certain direction. For long-span structures with N supports, the accelerations of ground motions at supports in the travelling direction can be expressed as the following N-dimensional vector

$$\ddot{\mathbf{u}}_b(t) = \{\ddot{u}_1(t), \ddot{u}_2(t), \cdots, \ddot{u}_N(t)\}^{\mathrm{T}} \tag{6}$$

At the same time, $\ddot{\mathbf{y}}_b(t)$ in Eq. (5) can also be expressed as the following m-dimensional ground acceleration vector

$$\ddot{\mathbf{y}}_b(t) = \{\ddot{y}_1(t), \ddot{y}_2(t), \cdots, \ddot{y}_m(t)\}^{\mathrm{T}} \tag{7}$$

Further, the transformation relation between $\ddot{\mathbf{y}}_b(t)$ and $\ddot{\mathbf{u}}_b(t)$ can be written as

$$\ddot{\mathbf{y}}_b(t) = \mathbf{E}_{mN}\ddot{\mathbf{u}}_b(t) \tag{8}$$

in which $\mathbf{E}_{mN}$ is an $m \times N$ block-diagonal matrix. Obviously, if no rotational components are considered for each support, then $m = 3N$.

It is assumed that α is the angle between the horizontal travelling direction of the seismic wave and the x-axis, which is defined as the longitudinal direction of the long structure. Hence for P waves, $\mathbf{E}_{mN}$ can be expressed as

$$\begin{bmatrix} \cos\alpha & 0 & \cdots & 0 \\ \sin\alpha & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ 0 & \cos\alpha & \cdots & 0 \\ 0 & \sin\alpha & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \cos\alpha \\ 0 & 0 & \cdots & \sin\alpha \\ 0 & 0 & \cdots & 0 \end{bmatrix} \tag{9}$$

while for SH and SV waves, each sub-matrix in $\mathbf{E}_{mN}$ becomes $\{-\sin\alpha \quad \cos\alpha \quad 0\}^{\mathrm{T}}$ and $\{0 \quad 0 \quad 1\}^{\mathrm{T}}$, respectively.

According to the transformation relation of Eq. (8), the right-hand term of Eq. (5) can be directly expressed by the ground acceleration at the support. Now, the equation of motion is similar to that of a uniform excitation earthquake, i.e.

$$\mathbf{M}_{aa}\ddot{\mathbf{y}}_d(t) + \mathbf{C}_{aa}\dot{\mathbf{y}}_d(t) + \mathbf{K}_{aa}\mathbf{y}_d(t) = \mathbf{R}\ddot{\mathbf{u}}_b(t) \tag{10}$$

in which

$$\mathbf{R} = \mathbf{M}_{aa}\mathbf{K}_{aa}^{-1}\mathbf{K}_{ab}\mathbf{E}_{mN} \tag{11}$$

3 Nonstationary random ground motion model with wave passage effect

The seismic ground motion is assumed to be a uniformly modulated nonstationary random process which is widely used in earthquake engineering. Considering the wave passage effect, i.e. the difference in the arrival times of waves, the ground accelerations at supports can be written as

$$\ddot{\mathbf{u}}_b(t) = \mathbf{G}(t)\ddot{\mathbf{x}}(t) \tag{12}$$

where

$$\mathbf{G}(t) = \mathrm{diag}[g(t-t_1), g(t-t_2), \cdots, g(t-t_N)], \;\; \ddot{\mathbf{x}}(t) = \begin{Bmatrix} \ddot{x}(t-t_1) \\ \ddot{x}(t-t_2) \\ \vdots \\ \ddot{x}(t-t_N) \end{Bmatrix} \tag{13}$$

in which $\mathbf{G}(t)$ is a diagonal matrix whose diagonal element $g(t)$ is a slowly varying modulation function and $\ddot{\mathbf{x}}(t)$ is a vector consisting of the stationary random process $\ddot{x}(t)$.

According to the Wiener-Khinchin theorem, the auto correlation function $R_{\ddot{x}\ddot{x}}(t_1 - t_2)$ of the stationary random process $\ddot{x}(t)$ can be expressed as

$$R_{\ddot{x}\ddot{x}}(t_1 - t_2) = E[\ddot{x}(t_1)\ddot{x}(t_2)] = \int_{-\infty}^{+\infty} S_{\ddot{x}\ddot{x}}(\omega)\mathrm{e}^{\mathrm{i}\omega(t_1-t_2)}\mathrm{d}\omega \tag{14}$$

where $S_{\ddot{x}\ddot{x}}(\omega)$ is the auto PSD function of $\ddot{x}(t)$.

Since the acceleration $\ddot{x}(t)$ is a stationary random process, the displacement $x(t)$ is also stationary. It has been proved [13] that the auto PSDs $S_{\ddot{x}\ddot{x}}(\omega)$ and $S_{xx}(\omega)$ and cross PSDs $S_{x\ddot{x}}(\omega)$ and $S_{\ddot{x}x}(\omega)$ satisfy the relationships

$$\begin{gathered} S_{xx}(\omega) = \frac{1}{\omega^4} S_{\ddot{x}\ddot{x}}(\omega) \\ S_{x\ddot{x}}(\omega) = S_{\ddot{x}x}(\omega) = -\frac{1}{\omega^2} S_{\ddot{x}\ddot{x}}(\omega) \end{gathered} \tag{15}$$

4 Frequency domain method for nonstationary random vibration analysis considering wave passage effect

4.1 Correlation analysis of random response

For a linear structure under the seismic excitation expressed in Eq. (12), the dynamic relative displacement vector can be written in the convolution integral form as follows

$$\mathbf{y}_d(t) = \int_{-\infty}^{+\infty} \mathbf{h}(\tau)\mathbf{R}\ddot{\mathbf{u}}_b(t-\tau)\mathrm{d}\tau \tag{16}$$

where $\mathbf{h}(\tau)$ is the impulse response function matrix. $\mathbf{h}(\tau)$ is related to the frequency response function matrix $\mathbf{H}(\omega)$ as a Fourier transform pair, i.e.

$$\mathbf{h}(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathbf{H}(\omega)\mathrm{e}^{\mathrm{i}\omega\tau}\mathrm{d}\omega,\ \ \mathbf{H}(\omega) = \int_{-\infty}^{\infty} \mathbf{h}(\tau)\mathrm{e}^{-\mathrm{i}\omega\tau}\mathrm{d}\tau \tag{17}$$

According to Eqs. (4) and (8), the quasi-static displacement $\mathbf{y}_s$ can be expressed as

$$\mathbf{y}_s(t) = -\mathbf{M}_{aa}^{-1}\mathbf{R}\mathbf{u}_b(t) \tag{18}$$

where $\mathbf{u}_b(t)$ is the displacement vector of the supports.

Substituting Eqs. (16) and (18) into Eq. (2) gives

$$\mathbf{y}_a(t) = \mathbf{y}_d(t) + \mathbf{y}_s(t) = \int_{-\infty}^{+\infty} \mathbf{h}(\tau)\mathbf{R}\ddot{\mathbf{u}}_b(t-\tau)\mathrm{d}\tau - \mathbf{M}_{aa}^{-1}\mathbf{R}\mathbf{u}_b(t) \tag{19}$$

It is noted that the first part of the right hand side of Eq. (19) is equivalent to a dynamic analysis with uniform excitation, while the second part is a linear transformation.

For a linear system with nonstationary random excitation, the random responses are also nonstationary. In order to assess the stochastic characteristics of random responses, a correlation analysis is performed based on random vibration theory. Multiplying each side of Eq. (19) by its transposition and performing an ensemble average gives

$$
\begin{aligned}
\mathrm{E}[\mathbf{y}_a(t_k)\mathbf{y}_{\boldsymbol{a}}^{\mathrm{T}}(t_l)] = \\
&\mathrm{E}\left[\mathbf{y}_d(t_k)\mathbf{y}_d^{\mathrm{T}}(t_l)\right] + \mathrm{E}\left[\mathbf{y}_s(t_k)\mathbf{y}_d^{\mathrm{T}}(t_l)\right] + \mathrm{E}[\mathbf{y}_d(t_k)\mathbf{y}_s^{\mathrm{T}}(t_l)] \\
&+\mathrm{E}[\mathbf{y}_s(t_k)\mathbf{y}_s^{\mathrm{T}}(t_l)]
\end{aligned}
\tag{20}
$$

Thus the autocorrelation function of the absolute displacement response $\mathbf{y}_a(t)$ consists of four parts which are the autocorrelation functions and cross-correlation functions of the dynamic relative displacement response $\mathbf{y}_d(t)$ and the quasi-static displacement response $\mathbf{y}_s(t)$.

In order to facilitate the derivation, the autocorrelation function of the dynamic relative displacement response $\mathbf{y}_d(t)$, i.e. the first term on the right hand side of Eq. (20), is studied first, and can be expressed as

$$
\begin{aligned}
\mathrm{E}\left[\mathbf{y}_d(t_k)\mathbf{y}_d^{\mathrm{T}}(t_l)\right] &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\mathbf{h}(\tau_k)\mathbf{R}\left(\mathrm{E}\left[\ddot{\mathbf{u}}_b(t_k-\tau_k)\ddot{\mathbf{u}}_b^{\mathrm{T}}(t_l-\tau_l)\right]\right)\mathbf{R}^{\mathrm{T}}\mathbf{h}^{\mathrm{T}}(\tau_l)\mathrm{d}\tau_k\mathrm{d}\tau_l \\
&= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\mathbf{h}(\tau_k)\mathbf{R}\mathbf{G}(t_k-\tau_k)(\mathrm{E}[\ddot{\mathbf{x}}(t_k-\tau_k)\ddot{\mathbf{x}}^{\mathrm{T}}(t_l-\tau_l)]) \\
&\qquad \mathbf{G}^{\mathrm{T}}(t_l-\tau_l)\mathbf{R}^{\mathrm{T}}\mathbf{h}^{\mathrm{T}}(\tau_l)\mathrm{d}\tau_k\mathrm{d}\tau_l
\end{aligned}
\tag{21}
$$

Thus the autocorrelation function of $\mathbf{y}_d(t)$ is related to the autocorrelation function of the stationary random acceleration vector $\ddot{\mathbf{x}}(t)$. To further simplify the results, setting $\bar{t}_k = t_k - \tau_k$ and $\bar{t}_l = t_l - \tau_l$ and applying the relation expressed in Eq. (14) gives

$$
\begin{aligned}
&\mathrm{E}[\ddot{\mathbf{x}}(t_k-\tau_k)\ddot{\mathbf{x}}^{\mathrm{T}}(t_l-\tau_l)] = \mathrm{E}[\ddot{\mathbf{x}}(\bar{t}_k)\ddot{\mathbf{x}}^{\mathrm{T}}(\bar{t}_l)] \\
&= \begin{bmatrix}
\mathrm{E}[\ddot{x}(\bar{t}_k-t_1)\ddot{x}(\bar{t}_l-t_1)] & \mathrm{E}[\ddot{x}(\bar{t}_k-t_1)\ddot{x}(\bar{t}_l-t_2)] \cdots & \mathrm{E}[\ddot{x}(\bar{t}_k-t_1)\ddot{x}(\bar{t}_l-t_n)] \\
\mathrm{E}[\ddot{x}(\bar{t}_k-t_2)\ddot{x}(\bar{t}_l-t_1)] & \mathrm{E}[\ddot{x}(\bar{t}_k-t_2)\ddot{x}(\bar{t}_l-t_2)] \cdots & \mathrm{E}[\ddot{x}(\bar{t}_k-t_2)\ddot{x}(\bar{t}_l-t_n)] \\
\vdots & \vdots \qquad \ddots & \vdots \\
\mathrm{E}[\ddot{x}(\bar{t}_k-t_n)\ddot{x}(\bar{t}_l-t_1)] & \mathrm{E}[\ddot{x}(\bar{t}_k-t_n)\ddot{x}(\bar{t}_l-t_2)] \cdots & \mathrm{E}[\ddot{x}(\bar{t}_k-t_n)\ddot{x}(\bar{t}_l-t_n)]
\end{bmatrix} \\
&= \int_{-\infty}^{\infty} \begin{bmatrix}
1 & \mathrm{e}^{\mathrm{i}\omega(t_1-t_2)} & \cdots & \mathrm{e}^{\mathrm{i}\omega(t_1-t_n)} \\
\mathrm{e}^{\mathrm{i}\omega(t_2-t_1)} & 1 & \cdots & \mathrm{e}^{\mathrm{i}\omega(t_2-t_n)} \\
\vdots & \vdots & \ddots & \vdots \\
\mathrm{e}^{\mathrm{i}\omega(t_n-t_1)} & \mathrm{e}^{\mathrm{i}\omega(t_n-t_2)} & \cdots & 1
\end{bmatrix} \mathrm{e}^{\mathrm{i}\omega(\bar{t}_k-\bar{t}_l)} S_{\ddot{x}\ddot{x}}(\omega)\mathrm{d}\omega \\
&= \int_{-\infty}^{\infty} \mathbf{W}^{*}\mathbf{e}\mathbf{e}^{\mathrm{T}}\mathbf{W}^{\mathrm{T}}\mathrm{e}^{\mathrm{i}\omega(\bar{t}_k-\bar{t}_l)} S_{\ddot{x}\ddot{x}}(\omega)\mathrm{d}\omega
\end{aligned}
\tag{22}
$$

where

$$
\mathbf{W} = \mathrm{diag}\left[\mathrm{e}^{-\mathrm{i}\omega t_1}, \mathrm{e}^{-\mathrm{i}\omega t_2}, \cdots, \mathrm{e}^{-\mathrm{i}\omega t_N}\right], \qquad \mathbf{e} = \begin{Bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{Bmatrix} \tag{23}
$$

Substituting Eq. (22) into Eq. (21), the auto correlation function of $\mathbf{y}_d(t)$ can be further expressed as

$$
\mathrm{E}\left[\mathbf{y}_d(t_k)\mathbf{y}_d^{\mathrm{T}}(t_l)\right] = \int_{-\infty}^{+\infty} \boldsymbol{\alpha}_d^{*}(t_k,\omega)\boldsymbol{\alpha}_d^{\mathrm{T}}(t_l,\omega)\mathrm{d}\omega \tag{24}
$$

where

$$
\begin{aligned}
\boldsymbol{\alpha}_d(t,\omega) &= \int_{-\infty}^{+\infty} \mathbf{h}(\tau)\tilde{\ddot{\mathbf{x}}}(t-\tau,\omega)\mathrm{d}\tau \\
\tilde{\ddot{\mathbf{x}}}(t,\omega) &= \mathbf{G}(t)\mathbf{W}\mathbf{e}\sqrt{S_{\ddot{x}\ddot{x}}(\omega)}\mathrm{e}^{\mathrm{i}\omega t}
\end{aligned}
\tag{25}
$$

The remaining three terms on the right hand side of Eq. (20) can be dealt in a similar way. For simplicity, their final expressions are given directly as follows:

(1) the auto correlation function of quasi - static displacement response $\mathbf{y}_s(t)$ can

be expressed as

$$
\begin{gathered}
\mathrm{E}[\mathbf{y}_s(t_k)\mathbf{y}_s^{\mathrm{T}}(t_l)] = \int_{-\infty}^{+\infty} \boldsymbol{\alpha}_s^*(t_k,\omega)\boldsymbol{\alpha}_s^{\mathrm{T}}(t_l,\omega)\mathrm{d}\omega \\
\boldsymbol{\alpha}_s(t,\omega) = -\mathbf{M}_{aa}^{-1}\mathbf{R}\tilde{\mathbf{x}} \\
\tilde{\mathbf{x}}(t,\omega) = \mathbf{G}(t)\mathbf{We}\sqrt{S_{xx}(\omega)}\mathrm{e}^{\mathrm{i}\omega t} = \frac{1}{\omega^2}\mathbf{G}(t)\mathbf{We}\sqrt{S_{\ddot{x}\ddot{x}}(\omega)}\mathrm{e}^{\mathrm{i}\omega t}
\end{gathered}
\tag{26}
$$

(2) the cross correlation function of dynamic relative displacement response $\mathbf{y}_d(t)$ and quasi - static displacement response $\mathbf{y}_s(t)$ can be expressed as

$$
\mathrm{E}[\mathbf{y}_d(t_k)\mathbf{y}_s^{\mathrm{T}}(t_l)] = \int_{-\infty}^{+\infty} \boldsymbol{\alpha}_d^*(t_k,\omega)\boldsymbol{\alpha}_s^{\mathrm{T}}(t_l,\omega)\mathrm{d}\omega \tag{27}
$$

(3) the cross correlation function of quasi - static displacement response $\mathbf{y}_s(t)$ and dynamic relative displacement response $\mathbf{y}_d(t)$ can be expressed as

$$
\mathrm{E}\big[\mathbf{y}_{\mathrm{s}}(t_k)\mathbf{y}_d^{\mathrm{T}}(t_l)\big] = \int_{-\infty}^{+\infty} \boldsymbol{\alpha}_{\mathrm{s}}^*(t_k,\omega)\boldsymbol{\alpha}_d^{\mathrm{T}}(t_l,\omega)\mathrm{d}\omega \tag{28}
$$

Using Eqs. (24) - (28) and setting $t_k = t_l = t$, the auto correlation function of the absolute displacement response $\mathbf{y}_a(t)$ can be expressed as

$$
\mathrm{E}[\mathbf{y}_a(t)\mathbf{y}_a^{\mathrm{T}}(t)] = \int_{-\infty}^{+\infty} (\boldsymbol{\alpha}_d(t,\omega) + \boldsymbol{\alpha}_s(t,\omega))^*(\boldsymbol{\alpha}_d(t,\omega) + \boldsymbol{\alpha}_s(t,\omega))^{\mathrm{T}}\mathrm{d}\omega \tag{29}
$$

According to the Wiener-Khinchin theorem, the integrand function on the right hand side of Eq. (29) is simply the PSD function of the absolute displacement response $\mathbf{y}_a(t)$, which is

$$\mathbf{S}_{y_a y_a}(t,\omega) = (\boldsymbol{\alpha}_d(t,\omega) + \boldsymbol{\alpha}_s(t,\omega))^* (\boldsymbol{\alpha}_d(t,\omega) + \boldsymbol{\alpha}_s(t,\omega))^{\mathrm{T}} \tag{30}$$

Then the time-dependent variance of absolute displacement response $\boldsymbol{y}_a(t)$ can be obtained as

$$\boldsymbol{\sigma}^2(t) = 2\int_0^{\infty} \mathbf{S}_{\mathrm{y}_a \mathrm{y}_a}(t,\omega)\mathrm{d}\omega \tag{31}$$

4.2 Frequency domain method for evolutionary PSD analysis

In the evolutionary PSD analysis of random responses of long-span structures, the dynamic relative displacement response $\mathbf{y}_d(t)$ is always calculated by using time domain methods. Hence, a small time step should be selected to achieve accurate results when high frequency components are involved in the excitation. However, the small time step makes the calculation inefficient. To solve this situation, a frequency domain method is presented for nonstationary vibration analysis of long-span structures. This method separates the deterministic and random vibration analyses and provides a semi-analytical solution for random responses with clear physical interpretations.

Applying the Fourier transform to $\ddot{\tilde{\mathbf{x}}}(t,\omega)$ in Eq. (25) gives

$$\begin{aligned}\ddot{\tilde{\mathbf{X}}}(\tilde{\omega},\omega) &= \int_{-\infty}^{+\infty} \ddot{\tilde{\mathbf{x}}}(t,\omega)\mathrm{e}^{-\mathrm{i}\tilde{\omega}t}\mathrm{d}t = \int_{-\infty}^{+\infty}\left(\mathbf{G}(t)\mathbf{W}(\omega)\mathbf{e}\sqrt{S_{\ddot{x}\ddot{x}}(\omega)}\mathrm{e}^{\mathrm{i}\omega t}\right)\mathrm{e}^{-\mathrm{i}\tilde{\omega}t}\mathrm{d}t \\ &= \widetilde{\mathbf{G}}(\tilde{\omega}-\omega)\mathbf{W}(\omega)\mathbf{e}\sqrt{S_{\ddot{x}\ddot{x}}(\omega)}\end{aligned} \tag{32}$$

where ω should be considered as a constant. The inverse transform of Eq. (32) can be expressed as

$$
\begin{aligned}
\ddot{\mathbf{x}}(t,\omega) &= \frac{1}{2\pi}\int_{-\infty}^{+\infty}\ddot{\tilde{\mathbf{X}}}(\tilde{\omega},\omega)\mathrm{e}^{\mathrm{i}\tilde{\omega}t}\mathrm{d}\tilde{\omega} \\
&= \frac{1}{2\pi}\int_{-\infty}^{+\infty}\tilde{\mathbf{G}}(\tilde{\omega}-\omega)\mathbf{W}(\omega)\mathbf{e}\sqrt{S_{\ddot{x}\ddot{x}}(\omega)}\mathrm{e}^{\mathrm{i}\tilde{\omega}t}\mathrm{d}\tilde{\omega}
\end{aligned}
\tag{33}
$$

where $\tilde{\mathbf{G}}(\tilde{\omega})$ is the Fourier transform matrix of $\mathbf{G}(t)$ and can be written as

$$
\tilde{\mathbf{G}}(\tilde{\omega}) = \int_{-\infty}^{+\infty}\mathbf{G}(t)\mathrm{e}^{-\mathrm{i}\tilde{\omega}t}\mathrm{d}t \tag{34}
$$

Combining Eq. (32) and (33), $\boldsymbol{\alpha}_{\mathrm{d}}(t,\omega)$ in Eq. (24) can be expressed as

$$
\boldsymbol{\alpha}_{\mathrm{d}}(t,\omega) = \boldsymbol{\beta}_{\mathrm{d}}(t,\omega)\sqrt{S_{\ddot{x}\ddot{x}}(\omega)}\mathrm{e}^{\mathrm{i}\omega t} \tag{35}
$$

where

$$
\boldsymbol{\beta}_{\mathrm{d}}(t,\omega) = \left(\frac{1}{2\pi}\int_{-\infty}^{+\infty}\mathbf{H}(\tilde{\omega}+\omega)\tilde{\mathbf{G}}(\tilde{\omega})\mathrm{e}^{\mathrm{i}\tilde{\omega}t}\mathrm{d}\tilde{\omega}\right)\mathbf{W}(\omega)\mathbf{e} \tag{36}
$$

It can be seen that the calculation of Eq. (36) is only related to $\tilde{\mathbf{G}}(\tilde{\omega})$, which is the Fourier transform matrix of the non-stationary random seismic input modulation function matrix $\mathbf{G}(t)$. The corresponding integral operation is equivalent to the inverse Fourier transform of the kernel function $\mathbf{H}(\tilde{\omega}+\omega)\tilde{\mathbf{G}}(\tilde{\omega})$, but note that the frequency corresponding to the frequency response function is $\tilde{\omega}+\omega$. The modulation function of uniformly modulated non-stationary seismic input is a slowly varying function, so the calculation does not need to use a very high sampling frequency. Also, this analysis process is deterministic, which has a good advantage for ~~fast Fourier transform~~ FFT.

Meanwhile, $\boldsymbol{\alpha}_{\mathrm{s}}(t,\omega)$ in Eq. (26) can be rewritten as

$$\begin{aligned}\boldsymbol{\alpha}_{\mathrm{s}}(t,\omega) &= \boldsymbol{\beta}_{\mathrm{s}}(t,\omega)\sqrt{S_{\ddot{x}\ddot{x}}(\omega)}\mathrm{e}^{\mathrm{i}\omega t}\\ \boldsymbol{\beta}_{\mathrm{s}}(t,\omega) &= -\frac{1}{\omega^2}\mathbf{M}_{\mathrm{aa}}^{-1}\mathbf{RG}(t)\mathbf{We}\end{aligned} \tag{37}$$

Substituting Eqs. (35) and (37) into Eq. (30), the evolutionary PSD of the absolute displacement response is given as

$$\mathbf{S}_{y_a y_a}(t,\omega) = (\boldsymbol{\beta}_d(t,\omega) + \boldsymbol{\beta}_s(t,\omega))^*(\boldsymbol{\beta}_d(t,\omega) + \boldsymbol{\beta}_s(t,\omega))^{\mathrm{T}} S_{\ddot{x}\ddot{x}}(\omega) \tag{38}$$

Thus, Eq. (38) gives the semi-analytical solution for the evolutionary PSD of random responses of long-span structures. This solution has a simple form and clear physical interpretations. It indicates that the nonstationary evolutionary PSD of the absolute displacement response is in fact an explicit modulation of the stationary PSD of the ground motion. Hence, when performing the similar nonstationary vibration analysis, it is only necessary to consider the calculation of the deterministic modulation matrix, i.e. $\boldsymbol{\beta}_d(t,\omega)$ and $\boldsymbol{\beta}_s(t,\omega)$ in Eqs. (36) and (37).

It should be mentioned that zero initial conditions are used in the above analysis. Compared to conventional time domain methods, the present method is totally implemented in the frequency domain. Since $\boldsymbol{\beta}_d(t,\omega)$ can be calculated by the FFT, a unified approach can be used for different type of modulation functions. Moreover, as well as the displacement calculated above, the evolutionary PSD of other random responses, such as the internal force, can also be solved by the present method without any additional difficulty.

4.3 Evaluation of extreme value response

The evaluation of the peak amplitude responses of long-span structures subjected to nonstationary seismic excitation is a fundamental problem for engineering structural design. In order to evaluate the extreme value responses, the nonstationary random response can be replaced with a stationary one through the energy equivalence over a specific duration T_d.

It is assumed that the evolutionary PSD $S_{yy}(t,\omega)$ of any random response $y(t)$ of a structure under non-stationary random earthquake is known. Over the duration T_d, the equivalent stationary PSD $S_{\bar{y}\bar{y}}(\omega)$ can be expressed as [13]

$$S_{\bar{y}\bar{y}}(\omega) = \frac{1}{T_d}\int_{t_0/\sqrt{2}}^{t_0/\sqrt{2}+T_d} S_{yy}(t,\omega)\mathrm{d}t \tag{39}$$

From the above equation, the PSD of the equivalent stationary random process $\bar{y}(t)$ with the average energy distribution, strong earthquake duration and seismic intensity consistent with the nonstationary stochastic process can be obtained. Denoting the extreme value of $\bar{y}(t)$ within the duration T_d as $\bar{y}_e$, and the standard deviation as $\sigma_{\bar{y}}$, a dimensionless parameter is defined as

$$\eta = \bar{y}_e/\sigma_{\bar{y}} \tag{40}$$

It is assumed that if a given threshold value is sufficiently high, the peaks of $\bar{y}(t)$

above this barrier will appear independently. Then, the number of crossings of the threshold value will be a Poisson process with a stationary increment [26]. Based on these assumptions, the probability distribution of η can be derived as

$$P(\eta) = \exp[-\nu T_d \exp(-\eta^2/2)] \tag{41}$$

where

$$\nu = \sqrt{\lambda_2/\lambda_0}/\pi \tag{42}$$

λ_0 and λ_2 are spectral moments of the random process and can be computed by

$$\lambda_k = 2\int_0^\infty \omega^k S_{\bar{y}\bar{y}}(\omega)\mathrm{d}\omega\,, k = 0,2 \tag{43}$$

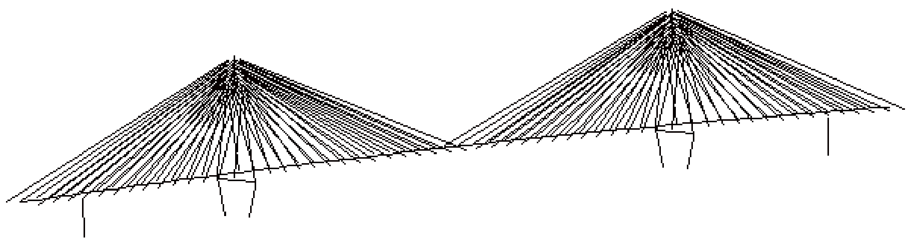
Using the probability distribution shown in Eq. (41), the expected value of η is approxmately

$$E(\eta) \approx \sqrt{2\ln(\nu T_d)} + \gamma/\sqrt{2\ln(\nu T_d)} \tag{44}$$

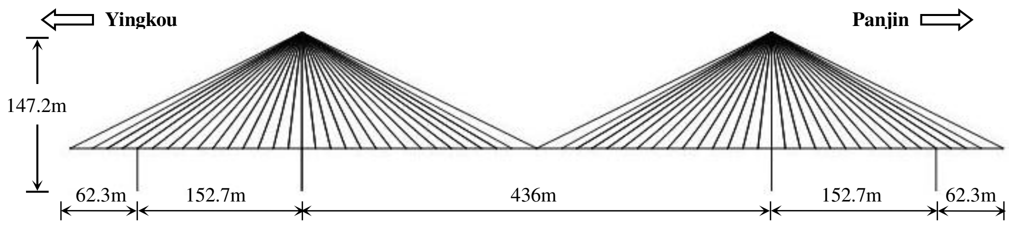
in which $\gamma = 0.5772$ is the Euler constant.

5 Numerical examples

The Liaohe bridge lying between Yinkou and Panjin in Liaoning Province, China is chosen as a numerical example, as shown in Fig. 1. The main structure spanning the Liao River is a cable-stayed bridge of total length 866m. The finite element model has 429 nodes (including 4 supports), 310 elements and 1156 DOF. The deck and tower are modelled by three dimensional beam elements with stiff arms on both ends and each cable is modelled by one dimensional cable elements.



(a) Oblique view



(b) Front view

Fig. 1 Schematic of the Liaohe bridge

The first 200 modes are used in the mode superposition, with the corresponding natural periods ranging within [0.046, 6.135]s. A damping ratio of 0.05 is assumed for all participant modes. The effective frequency region is taken as $\omega \in [0.0,100]$rad/s and the frequency step size is $\Delta\omega = 0.2$rad/s. The ground acceleration response spectrum used is based on the Chinese code (CMC, 2001) [27] with regional fortification intensity 7, site-type 2, and seismic classification 1. The Kaul method [28] is used to generate the ground acceleration PSD compatible with the response spectrum.

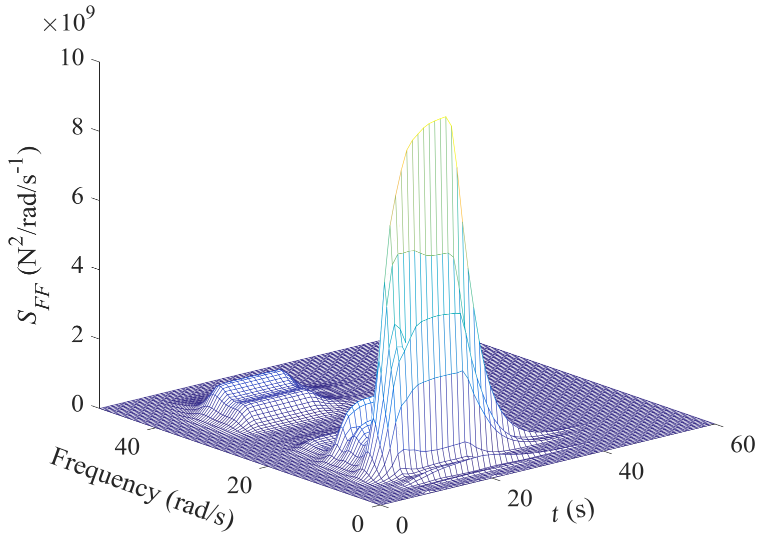
A uniformly modulated nonstationary seismic excitation model is used here, with the modulation function

$$a(t)=\begin{cases} I_0(t/t_1)^2 & 0\le t\le t_1 \\ I_0 & t_1\le t\le t_2 \\ I_0\exp[c_0(t-t_2)] & t\ge t_2 \end{cases} \tag{45}$$

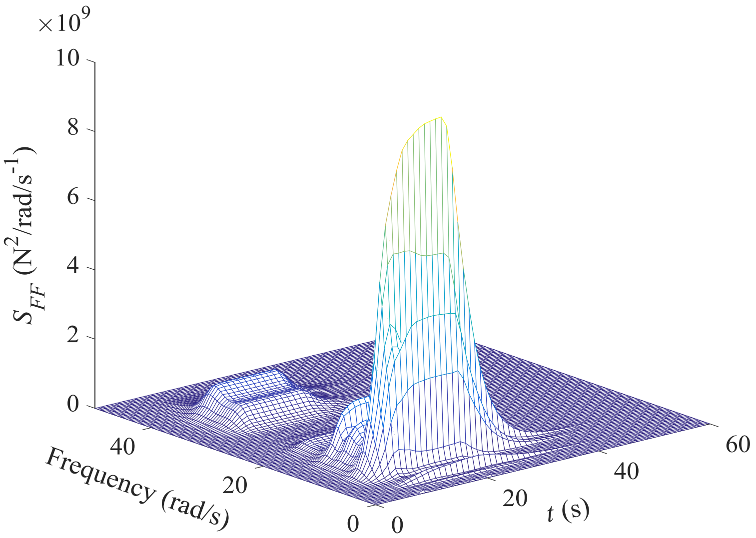
where $t_1=8.0\,\mathrm{s}$, $t_2=20.0\,\mathrm{s}$ and $c=0.2$. The duration of the earthquake is $t\in[0{,}60\mathrm{s}]$.

5.1 Evolutionary PSD and time-dependent variance

The PEM [24] is used to benchmark the results obtained from the present method. The SV waves travelling horizontally along the bridge are considered as the excitation and the wave velocity is v =2000m/s. A time step with Δt =0.02s is used in the time domain analysis of the PEM, while a sampling frequency $f=10\mathrm{Hz}$ is used in the FFT of the present method. Figs. 2(a) and 2(b) show the evolutionary PSD functions of the



(a) Results of the PEM



(b) Results of the present method

Fig. 2 Evolutionary PSD of transverse shear force at the middle of the deck

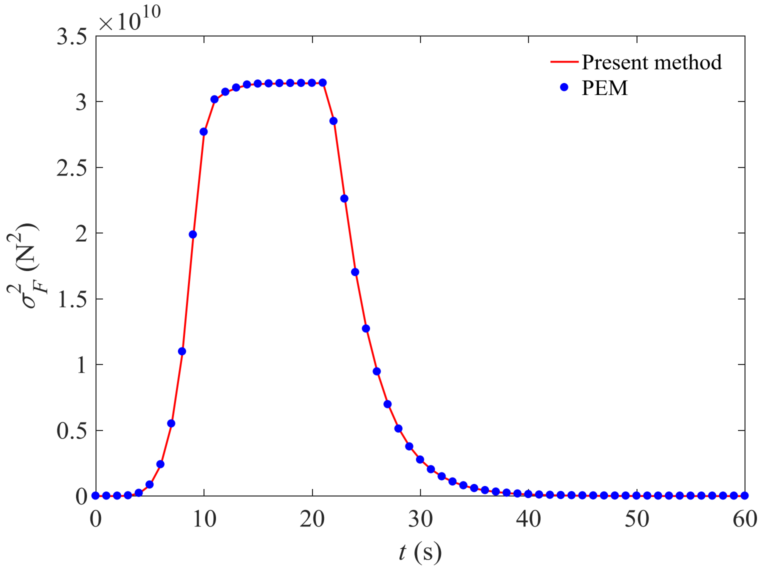
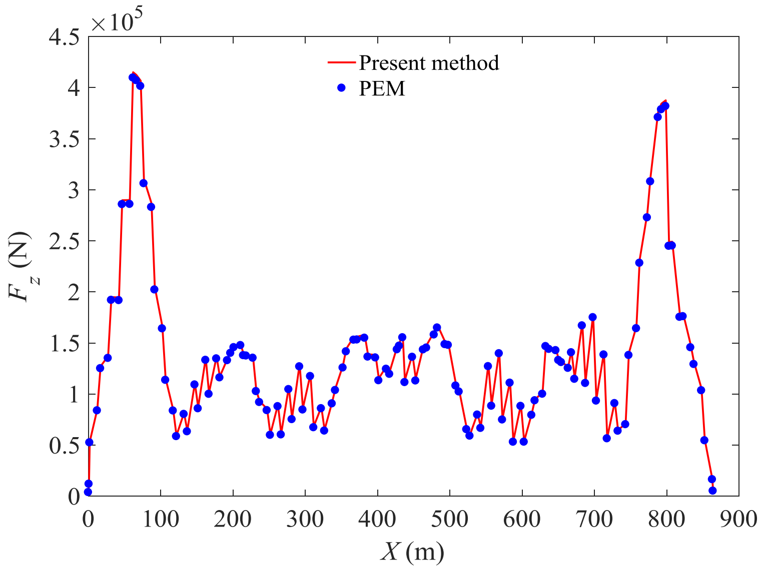


Fig. 3 Time-dependent variances of transverse shear force at the middle of the deck.

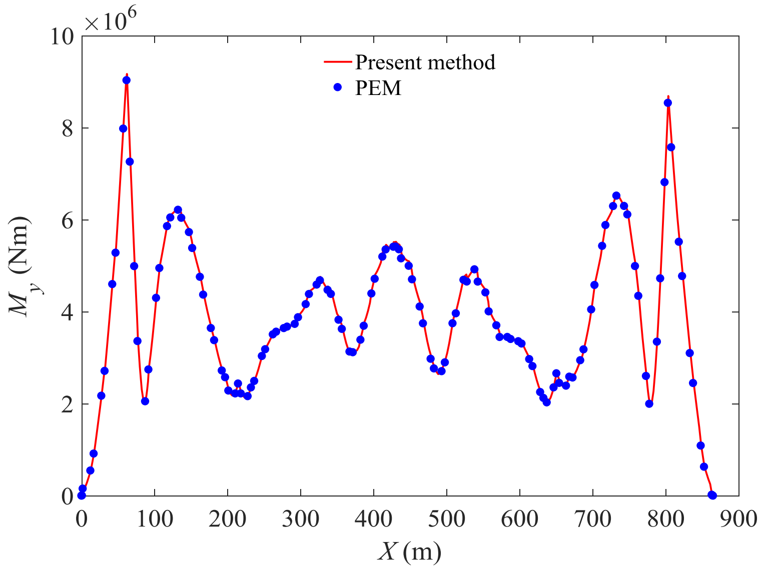
transverse shear force at the middle of the deck obtained from the PEM and present method, respectively. It is observed that the results of these two methods agree quite well and the maximum error is about 0.76%. For further comparison, Fig. 3 gives the time-dependent variances of the transverse shear force at the middle of the deck. It is seen that the results obtained by the present method are in excellent agreement with those of the PEM. The maximum relative error is below 0.4%, and thus the accuracy of the present method is verified.

5.2 Extreme value response

Considering P waves with wave velocity $v = 1000$m/s and SV waves with $v = 700$m/s, extreme value responses of the bridge are estimated by the present method and PEM. Figs. 4(a) and 4(b) present extreme values of the transverse shear forces F_z

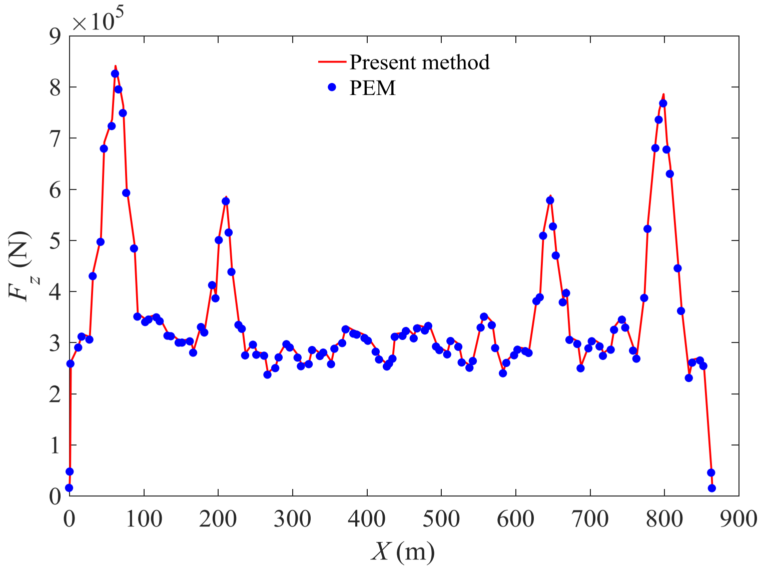


(a) Transverse shear forces

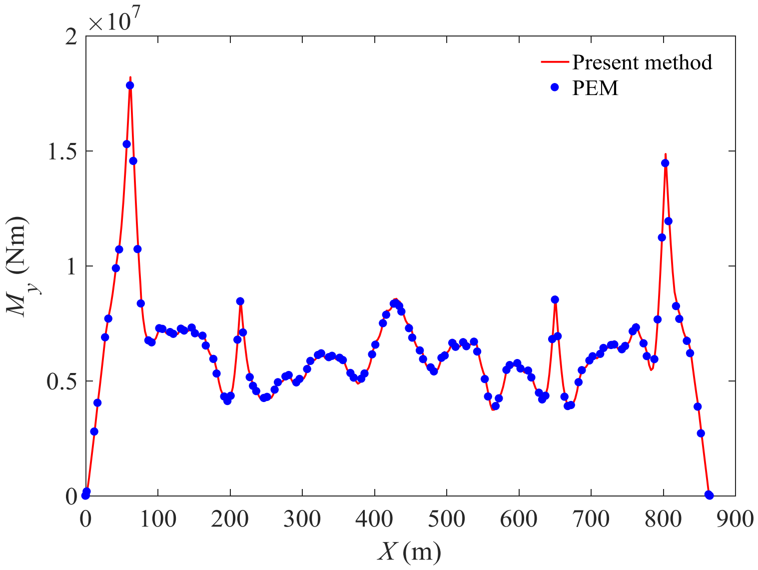


(b) Bending moments

Fig. 4 Extreme value responses of internal forces under P waves



(a) Transverse shear forces

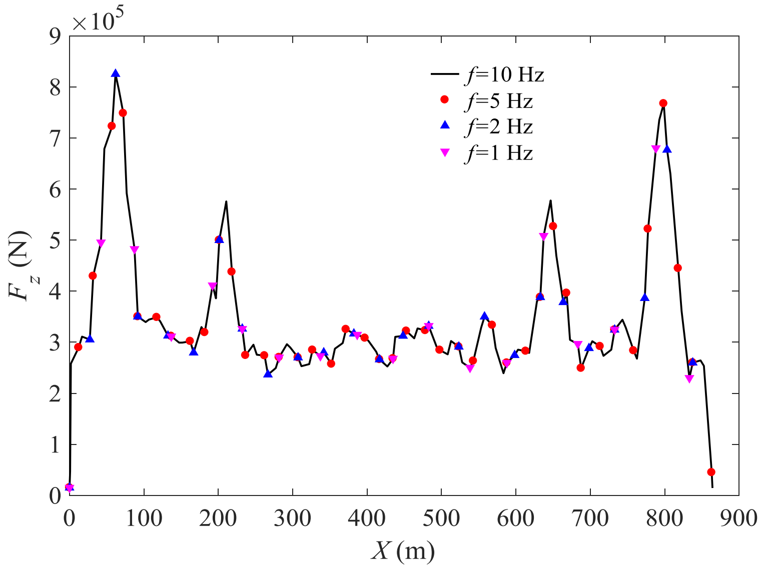


(b) Bending moments

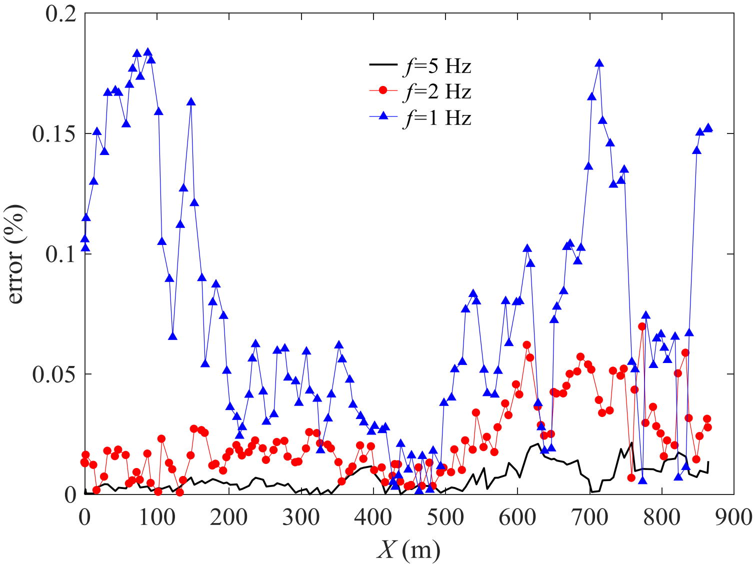
Fig. 5 Extreme value responses of internal forces under SV waves and bending moments $\boldsymbol{M_y}$ along the deck under P waves, respectively, while Figs. 5(a) and 5(b) present the same results under SV waves. It is shown that the results using the present method and PEM have a good agreement, demonstrating the accuracy of the present method for extreme value responses. As can be seen from Fig. 4(a), there are two peak values of transverse shear forces at $\boldsymbol{X} =$ 62m and 803m, i.e. the locations of the left and right bridge piers. This is because the restraints of piers can change the distribution of internal forces and lead to jumps of transverse shear forces. Between these two piers, the distribution of transverse shear forces is comparatively flat. Moreover, due to the symmetry of the bridge and excitation, the overall distribution of transverse shear forces also shows approximate symmetry. Similar phenomena can be observed in Figs. 4(b), 5(a) and 5(b), respectively. Computation times of the present method and PEM are 667.52s and 1430.15s, indicating the high efficiency of the present method.

5.3 Performance of the present method with different sampling frequencies

In Section 4.2, it was pointed out that for a linear system under uniformly modulated non-stationary random seismic loads, the evolutionary PSD of the response is determined by Eq. (38), and its physical meaning is the evolution modulation of the input stationary stochastic process, which can be determined by the coefficient vectors $\boldsymbol{\beta}_d(t,\omega)$ and $\boldsymbol{\beta}_s(t,\omega)$. For the calculation of $\boldsymbol{\beta}_d(t,\omega)$ by Eq. (36), only the frequency domain transform of the input nonstationary random process modulation is needed. Since slowly varying modulation functions are used to represent the nonstationary characteristic of the ground motion, a small sampling frequency can be used in the FFT to reduce the computational cost. To demonstrate this advantage, the present method is implemented with different sampling frequencies, i.e. $f = 10\text{Hz}$, 5Hz, 2Hz and 1Hz. The extreme transverse shear forces of the bridge under SV waves with v =700m/s is shown in Fig. 6(a). It is seen that results with different sampling frequencies are almost coincident with each other. For the convenience of comparison, the result with sampling frequency $f = 10\text{Hz}$ is employed as a reference solution, and then relative errors of results with smaller sampling frequencies are given in Fig. 6(b). It can be seen that maximum errors of results with $f = 1\text{Hz}$, 2Hz and 5Hz are respectively 0.2%, 0.05% and 0.025%. Similar to Fig. 6, Fig. 7 shows results for the extreme bending moment with different sampling frequencies. As can be seen from Fig. 7(b), the maximum errors of results with $f = 1\text{Hz}$, 2Hz and 5Hz are respectively 0.25%, 0.1% and 0.025%. The computation times corresponding to different sampling frequencies are shown in Table 1. It is observed that the computation time for $f = 1\text{Hz}$ is 284.18s, which is about 40% of that for $f = 10\text{Hz}$. Thus, from the results above, it appears that the present method can be implemented with a very small sampling frequency while retaining very high accuracy, and hence its computational efficiency is improved significantly.

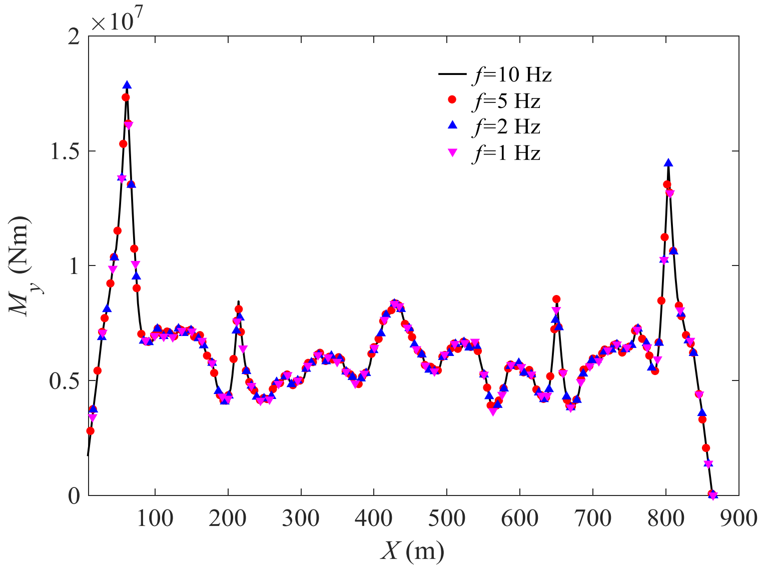


(a) Transverse shear forces

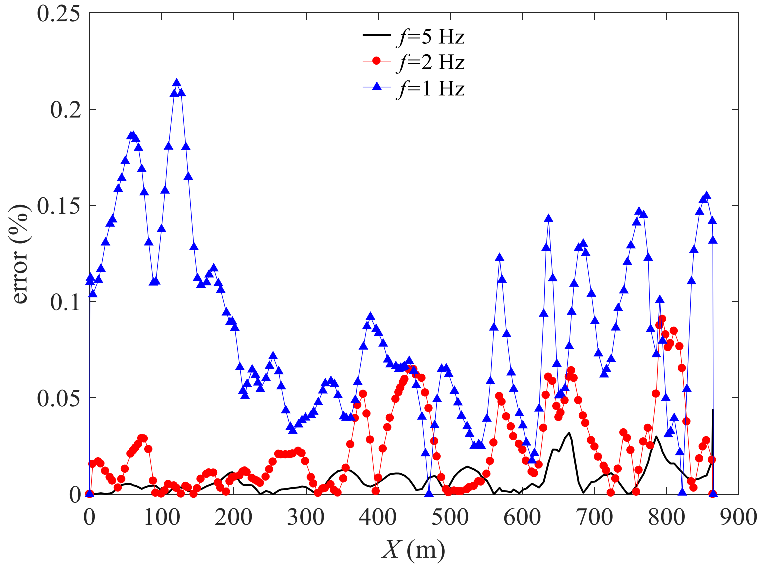


(b) Relative error

Fig. 6 Extreme value transverse shear forces with different sampling frequencies



(a) Bending moment



(b) Relative error

Fig. 7 Extreme value bending moments with different sampling frequencies

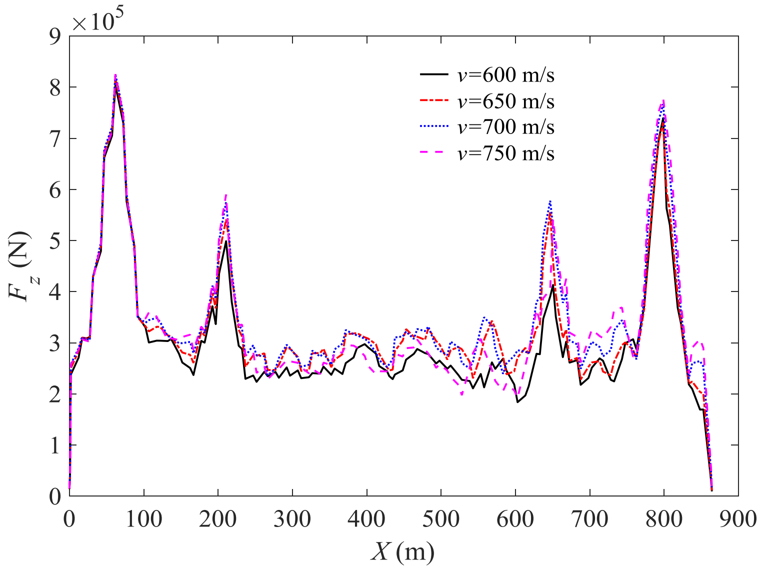
Table 1 Computation times of the present method with different sampling frequencies

Sampling frequencies (Hz)	10	5	2	1
Time (s)	667.52	311.65	305.69	284.18

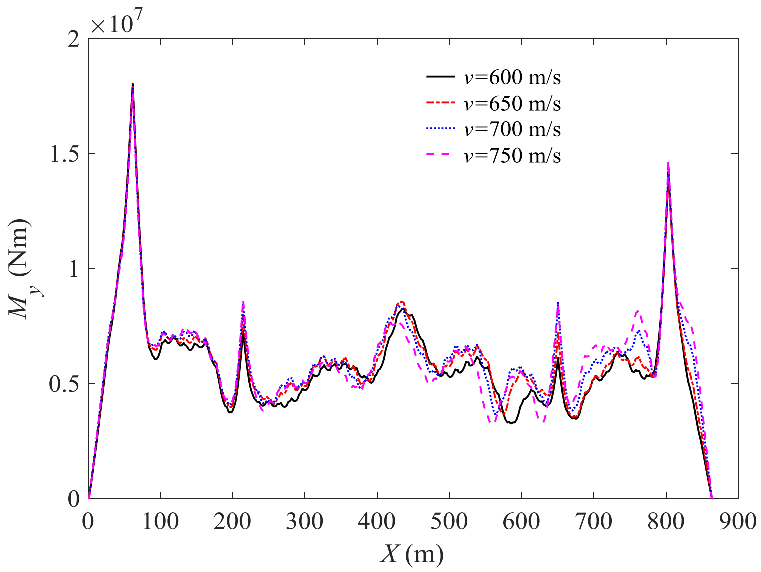
5.4 Influences of the wave passage effect on responses

Influences of the wave passage effect on random seismic responses are investigated. ~~Consider the response of the structure under SV waves propagating along the longitudinal direction of the bridge with velocities v =600m/s，650m/s，700m/s and 750m/s. The modal number, frequency domain analysis parameters and nonstationary seismic models are the same as above.~~ SV waves propagating along the longitudinal direction of the bridge with velocities v =600m/s，650m/s，700m/s and 750m/s are considered for the seismic response of the bridge, while the modal number, frequency domain analysis parameters and nonstationary seismic models are the same as above. The frequency domain analysis method proposed in this paper is used with sampling frequency $f =$ 2Hz. Fig. 8(a) gives transverse shear forces with different wave velocities. It is observed that, as the wave velocity increases, these differ slightly outside the two side piers, i.e. in the ranges 0 to 62.3m and 803.7 to 866m, but differ significantly between these two piers, i.e. in the range 62.3 to 803.7m.

According to random vibration analysis of the structure under multi-input nonstationary seismic excitation in Section 4, the absolute displacement response of the structure is generated by the dynamic relative displacement response and the quasi-static displacement response. In fact, the long-span cable-stayed bridge can be regarded as a complex floating system, and the force transmission path is the main deck drawn by the cable, and passed to the bridge tower, and then passed to the foundation. At the same time the deck also is restrained by the two side piers. Considering the wave effect of seismic propagation, the quasi-static displacement caused by the non-uniform motion of the supports has a significantly higher effect on the shear force of the deck between the two side piers. Fig. 8 (b) shows the results of the calculation of the bending moment of the main deck under different wave velocities. Similar phenomena are observed to those of the shear response. In addition, it can be seen from Figs. 8 (a) and 8 (b) that there is no obvious law for the variation of the peak value of the response, which is influenced by the quasi-static displacement response and the dynamic relative displacement response. For a complex structure, it is often difficult to determine which type of vibration mode has a major effect on its seismic response, and the apparent wave velocity obtained under different earthquakes is often very different. In engineering practice, in the absence of sufficiently reliable wave velocity measurement data, it is appropriate to select the most unfavorable situation as a design basis.



(a) Transverse shear forces



(b) Bending moments

Fig. 8 Internal forces with different wave velocities

6 Conclusions

This paper presents a frequency domain method for the seismic response analysis of long-span structures subjected to nonstationary random ground motions with consideration of the wave passage effect. A semi-analytical solution is derived for the evolutionary PSD of the response. The following conclusions can be drawn:

(1) The nonstationary evolutionary PSD of responses can be represented explicitly as the modulation of the stationary PSD of the ground motion, while the corresponding modulation matrix can be obtained from the nonstationary modulation function. For slowly varying modulation functions, a small sampling frequency can be used in the FFT and hence the present method gains its high efficiency.

(2) The results presented for a cable-stayed bridge show that the wave passage effect has significant influence on the random response and hence should be considered in the seismic analysis of long-span structures. The actual seismic response is determined by the dynamic relative displacement and the quasi-static displacement. When seismic analysis is carried out for a multiply supported structure, the influence of the wave passage effect should be taken into account.

(3) Since the wave passage effect of ground motions is considered, supports of long-span structures will motion in different phases, which may result two further effects, i.e., the non-uniform dynamic subsidence of supports and the cancellation of inertia forces. These two effects have opposing influences on dynamic responses of long-span structures.

Hence, it is possible for the responses to be larger or smaller after considering the wave passage effect, and these changes cannot be determined a priori. In practical engineering, in the absence of sufficiently reliable wave velocity measurement data, it is recommended to perform a series of seismic analyses with different wave velocities and then select the most unfavorable situation as a basis for design.

Acknowledgments

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